Stochastic Finite-Element Analysis of Seismic Soil–Structure Interaction

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Abstract: A procedure is presented for the probabilistic analysis of the seismic soil-structure interaction problem. The procedure accounts for uncertainty in both the free-field input motion as well as in local site conditions, and structural parameters. Uncertain parameters are modeled using a probabilistic framework as stochastic processes. The site amplification effects are accounted for via a randomized relationship between the soil shear modulus and damping on the one hand, and the shear strain of the subgrade on the other hand, as well as by modeling the shear modulus at low strain level as randomly fluctuating with depth. The various random processes are represented by their respective Karhunen-Loève expansions, and the solution processes, consisting of the accelerations and generalized forces in the structure, are represented by their coordinates with respect to the polynomial chaos basis. These coordinates are then evaluated by a combination of weighted residuals and stratified sampling schemes. The expansion can be used to carry out very efficiently, extensive Monte Carlo simulations. The procedure is applied to the seismic analysis of a nuclear reactor facility.

DOI: 10.1061/(ASCE)0733-9399(2002)128:1(66)

CE Database keywords: Finite-element method; Soil-structure interaction; Seismic response; Stochastic processes.

Introduction

The reassessment of existing massive commercial and defense hazardous facilities is a high priority activity involving major engineering challenges. Usually, hazardous facilities include heavy and stiff concrete structures, partially embedded, embedded, or even buried the dynamical response of which is significantly affected by their interaction with the surrounding soil. Dynamic soil-structure interaction (SSI) is a complex phenomenon with significant uncertainties associated with the input motions as well as the analytical models used for both the interaction and the dynamical properties of the materials involved. The ability to rationally account for these uncertainties and propagate their effects to the predicted behavior of the associated systems has the potential of enhancing their reliability and reducing the cost associated with their maintenance.

The present paper applies techniques from stochastic finite elements (Ghanem and Spanos 1991) to the probabilistic characterization and probabilistic risk assessment (PRA) of hazardous facilities under dynamic loads associated with such extreme events as strong ground motions. The primary goal of this article is to enhance current analysis techniques of such facilities by demonstrating how these stochastic finite element formulations can be used to integrate models of uncertainty with state-of-the-art methods in seismic soil-structure interaction. Uncertainties in a number of key factors can be simultaneously accounted for. The method used in this article permits the efficient simulation of response quantities that are consistent with specific probabilistic models of the input data. The novelty of the approach lies in its ability to handle a combination of sources for the uncertainty, modeled in a probabilistic framework. The paper also extends current stochastic finite element techniques to coupled stochastic systems and to systems featuring nonlinear stochastic constitutive behavior. Furthermore, a new procedure is developed and implemented in the paper for the convergence acceleration of the polynomial chaos expansions used to approximate the stochastic solution.

Extensive studies on probabilistic SSI have been carried out at the Lawrence Livermore National Laboratory (LLNL) (LLNL 1993) and the Brookhaven National Laboratory (BNL) (Pires et al. 1985). The LLNL study was based on a large number of case studies with the aim of identifying the most significant variables for seismic SSI effects and their influence on structural response variability. However, the LLNL study did not involve any methodology for characterizing probabilistically either the input data or the model predictions. The BNL study focused on nuclear containment structures using the linear random vibration theory to calculate limit state probabilities under random seismic loads. The BNL methodology is restricted to superficial rigid circular foundations on a viscoelastic half-space. For realistic situations including arbitrarily shaped and/or flexible foundations, partially embedded or buried structures, oblique seismic waves, and nonuniform soil layering, the BNL methodology is not directly applicable. The stochastic approach presented in the present paper addresses these aspects. Moreover, compared with the current lognormal format used in most PRA methodologies for hazardous facilities (Reed and Kennedy 1994), the stochastic procedure used in this paper produces considerably more accurate results for fragility analyses.

Specifically, by specifying the free-field input motion as a random process with spatial random fluctuations, care is taken in producing input motion records that are commensurate with

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Note. Associate Editor: Ahsan Kareem. Discussion open until June 1, 2002. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on January 23, 2001; approved on February 12, 2001. This paper is part of the ***Journal of Engineering Mechanics***, Vol. 128, No. 1, January 1, 2002. ©ASCE, ISSN 0733-9399/2002/1-66–77/$8.00+$.50 per page.

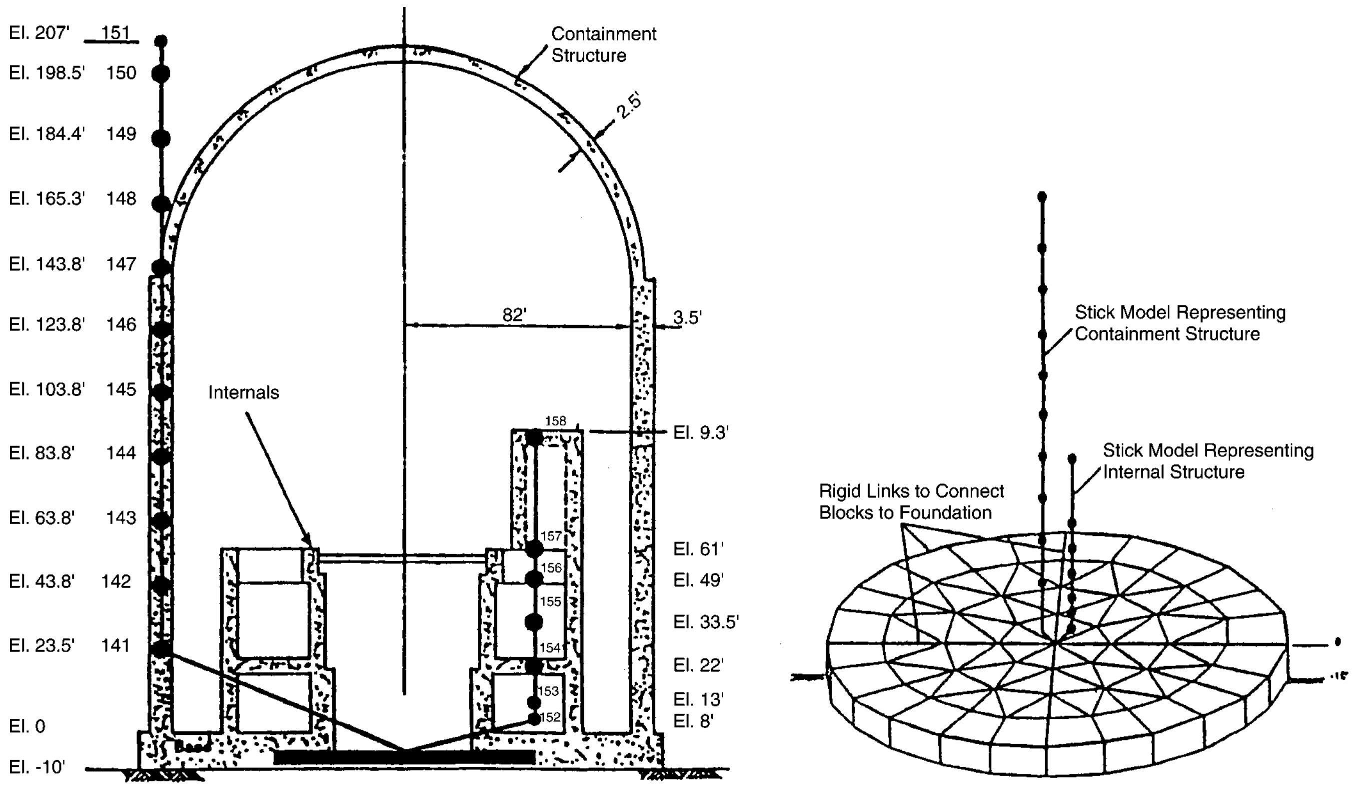

Fig. 1. Structural model of reactor building

specified probabilistic site response spectra. The probabilistic character of these spectra features frequency-dependent fluctuations and thus alleviates a number of constraints associated with the traditional lognormal assumption. Additionally, the shear modulus and material damping are assumed to be randomly varying functions of the shear strain, with this variability being also represented as a random field. A significant role in describing the dependence of shear modulus on shear strain is played by the value of the shear modulus at small levels of strain. This quantity is assumed to be varying randomly with depth, thus effectively inducing a random layering of the medium that can be construed as characterizing local site conditions. The random processes modeling the input motion, the low strain shear modulus, and the dependence of shear modulus on shear strain are then used as input to a site response analysis program which computes the actual soil motion in the free field (Idriss and Sun 1991). The output from this site response analysis is then used as input to an industry-standard software package for soil-structure interaction analysis (Lysmer et al. 1988) that has been enhanced to accommodate stochastic models for the structural stiffness and damping (Ghiocel 1996b). The superstructure is represented by two stick beams modeling the containment structure of a nuclear power plant and its internal structure, respectively, as shown in Fig. 1. The effective modulus of elasticity and material damping of the containment structure are assumed to be random variables with specified means and standard deviations.

In the next section, the equations governing the motion of the soil-structure system are briefly reviewed. Following that, two expansions are introduced that are used in the sequel for representing random processes. These are the Karhunen-Loève and the polynomial chaos expansions. The stochastic finite element method is then briefly reviewed, and some specific details about its implementation are presented. Probabilistic models for the various random quantities are then introduced, and finally, a numerical example is presented that exemplifies the proposed approach.

Equations of Motion

The computer program ACS-SASSI (Ghiocel 1996b) is used for both the site response and the soil-structure analyses. This computer package implements the flexible volume substructuring method (Lysmer et al. 1988) formulated in the frequency domain using the complex response function and finite element analysis. Accordingly, the complete soil-structure system (Fig. 2) is partitioned into two substructures, namely the foundation and the structure, as shown in Figs. 2(b) and (c), respectively. In this partitioning, the structure consists of the superstructure plus the foundation minus the excavated soil. Interaction between the structure and the foundation occurs at all foundation nodes. The equations of motion are given by,

$$\begin{bmatrix} \mathbf{C}_{ss} & \mathbf{C}_{si} \\ \mathbf{C}_{is} & (\mathbf{C}_{ii}-\mathbf{C}_{ff}+\mathbf{X}_{ff}) \end{bmatrix} \begin{Bmatrix} \mathbf{u}_s \\ \mathbf{u}_f \end{Bmatrix} = \begin{Bmatrix} 0 \\ \mathbf{X}_{ff}\mathbf{u}_f^i \end{Bmatrix} + \begin{Bmatrix} \mathbf{P}_{xs} \\ \mathbf{P}_{xf} \end{Bmatrix} = \begin{Bmatrix} \mathbf{P}_s \\ \mathbf{P}_f \end{Bmatrix} \quad (1)$$

from which the final total motion of the structure can be determined. In these equations, the subscripts s, i, and f refer to degrees of freedom associated with the nodes on the superstructure, foundation, and excavated soil, respectively. The matrix $\mathbf{C}$ is the complex frequency-dependent stiffness matrix given by,

$$\mathbf{C}(\omega)=\mathbf{K}-\omega^2\mathbf{M} \quad (2)$$

where $\mathbf{M}$ and $\mathbf{K}$ denote the total mass and complex stiffness matrices, respectively, and ω is the frequency of vibration. Furthermore, $\mathbf{u}$ denotes the complex Fourier coefficients of the modal displacement solution, and $\mathbf{X}_{ff}$ is the complex impedance matrix,

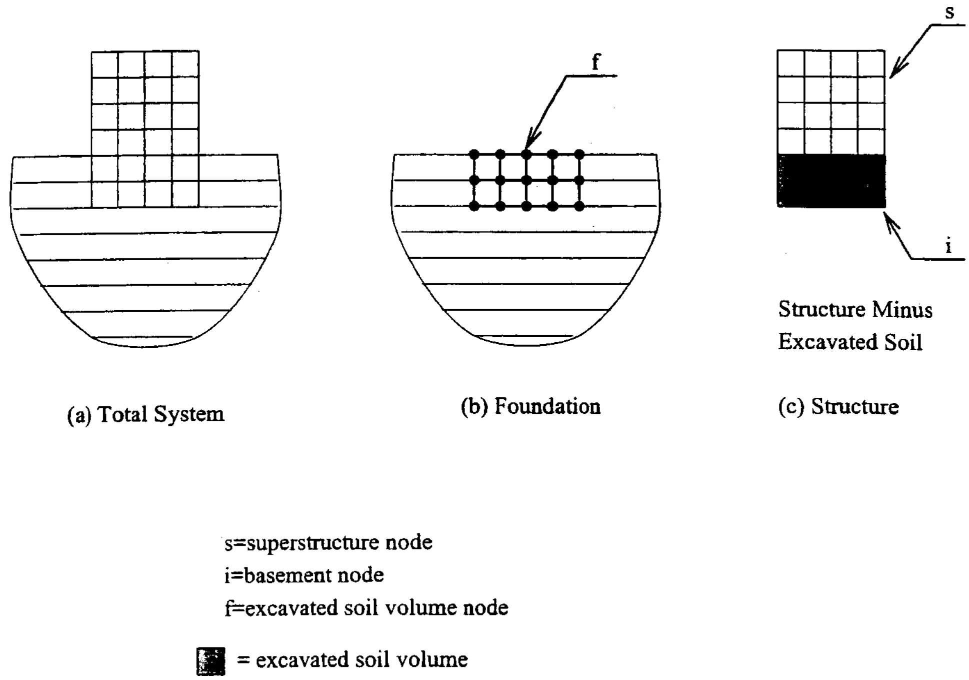

Fig. 2. Soil-structure interaction substructuring

which is a frequency dependent matrix representing the dynamic stiffness of the foundation at the interaction nodes. Moreover, $\mathbf{P}_{xs}$ and $\mathbf{P}_{xf}$ are the amplitudes of the external forces at the superstructure and foundation nodes, respectively, with $\mathbf{P}_s$ and $\mathbf{P}_f$ denoting the net forces at those nodes.

The standard solution of this problem involves three main steps, performed at each frequency. First, the site response problem is solved to determine the free field motion $\mathbf{u}_f$ driving the embedded part of the structure. Second, the impedance problem is solved to determine the matrix $\mathbf{X}_{ff}$. Finally, the structural problem is solved for the nodal displacements inside the structure.

Computational Representation of Stochastic Fields

In the proposed approach, the key idea is to provide a global description of the response surface as a function of a denumerable set of random variables. The implementation of the proposed approach is achieved in two steps. The first one involves an expeditious condensation of the basic random processes via the Karhunen-Loève expansion. The second step evaluates the coefficients of a stochastic orthogonal polynomial expansion of system response. After the coefficients of this polynomial expansion have been calculated, points on the system response surface can be readily simulated to evaluate probabilities of various events of interest.

Stochastic Representation of Dynamic Loading and System Parameters

A major concern of the present paper deals with the characterization of the prediction from a model of a physical phenomenon where some parameters of the model have been represented as stochastic processes. The answer to this question draws on an analogy from the deterministic approximation theory, where a solution to a problem is identified with its projection on a basis in an appropriate function space. It often happens, in deterministic analysis, that the coefficients in such a representation have an immediate physical meaning, which distracts from the mathematical significance of the solution. Carrying this argument over to the case involving stochastic processes, the solution to the problem is identified with its projection on a set of appropriately chosen basis functions. A random variable, is therefore viewed as a function of a variable, θ, that refers to the space of elementary events. As functions, random variables define a Hilbert space in which approximations are sought. The first step in that effort is to identify suitable bases, two of which are introduced in this section.

Karhunen-Loève Expansion

The Karhunen-Loève expansion of a stochastic process $p(\mathbf{x},\theta)$, is based on the spectral expansion of its covariance function $R_{pp}(\mathbf{x},\mathbf{y})$. Here, the argument θ indicates the random nature of the corresponding quantity, while $\mathbf{x}$ and $\mathbf{y}$ are used to denote elements of the indexing set, which in the present context refers to the spatial extent of the problem. The covariance function being symmetrical and positive definite, by definition, has all its eigenfunctions mutually orthogonal, and they form a complete set spanning the function space to which $p(\mathbf{x},t)$ belongs. It can be shown that if this deterministic set is used to represent the process $p(\mathbf{x},\theta)$, then the random coefficients used in the expansion are also orthogonal. The expansion then takes the following form

$$p(\mathbf{x},\theta)=\langle p(\mathbf{x})\rangle+\sum_{i=1}^{\infty}\sqrt{\lambda_i}\xi_i(\theta)\phi_i(\mathbf{x}) \tag{3}$$

where $\langle\cdot\rangle$ denotes the operator of mathematical expectation, and $\{\xi_i(\theta)\}$ forms a set of zero-mean orthonormal random variables. Furthermore, $\{\phi_i(\mathbf{x})\}$ are the normalized eigenfunctions and $\{\lambda_i\}$ are the eigenvalues, of the covariance kernel, and can be evaluated as the solution to the following integral equation:

$$\int_{\mathcal{D}} R_{pp}(\mathbf{x},\mathbf{y})\phi_i(\mathbf{y})d\mathbf{y}=\lambda_i\phi_i(\mathbf{x}) \tag{4}$$

where $\mathcal{D}$ denotes the spatial domain over which the process $p(\mathbf{x},\theta)$ is defined. The most important aspect of this spectral representation is that the spatial random fluctuations have been decomposed into a set of deterministic functions in the spatial variables multiplying random coefficients that are independent of these variables. If the random process being expanded, $p(\mathbf{x},\theta)$, is Gaussian, then the random variables $\{\xi_i\}$ form an orthonormal Gaussian vector. The Karhunen-Loève expansion is mean-square convergent irrespective of the probabilistic structure of the process being expanded, provided it has a finite variance. The monotony of the decay in the magnitude is guaranteed by the symmetry of the covariance function, and the rate of the decay is inversely proportional to the correlation length of the process being expanded. Thus, the closer a process is to white noise, the more terms are required in its expansion, while at the other limit, a random variable can be represented by a single term. In physical systems, it can be expected that material properties vary smoothly at the scales of interest to most applications, and therefore only a few terms in the Karhunen-Loève expansion can capture most of the uncertainty in the process. It should be emphasized that in comparison with other mathematical representations, the Karhunen-Loève expansion requires the minimum number of terms for a specified accuracy.

Of particular interest in the present analysis is the probabilistic modeling of positive random fields such as the amplitude of an input motion as a function of frequency or of soil stiffness and hysteretic damping profiles as a function of depth, both being positive quantities. Lognormal random fields, defined as the exponentials of some appropriate gaussian fields, are used to encapsulate the probabilistic variability in these quantities. It can be shown that the correlation function of a lognormal field $l(\mathbf{x},\theta)$ is related to that of its associated gaussian field, $g(\mathbf{x},\theta)$, through the relation (Ghanem 1999a, b),

$$\langle \tilde{g}(\mathbf{x}_i,\theta)\tilde{g}(\mathbf{x}_j,\theta)\rangle = \ln(1+[\langle \tilde{l}(\mathbf{x}_i,\theta)\tilde{l}(\mathbf{x}_j,\theta)\rangle]/[\langle l(\mathbf{x},\theta)\rangle]) \tag{5}$$

where a tilde refers to the demeaned process. Thus, given the statistics of the lognormal field, the statistics of the associated normal field are first calculated, its Karhunen-Loève expansion is computed, and realizations of the lognormal random field are obtained by exponentiation according to

$$l(\mathbf{x},\theta) = e^{g(\mathbf{x},\theta)} = \exp\left[\langle g(\mathbf{x},\theta)\rangle + \sum_{i=1}^{\infty} \sqrt{\lambda_i}\xi_i(\theta)\phi_i(\mathbf{x})\right] \tag{6}$$

Expanding each of the random quantities in their respective Karhunen-Loève expansion, each random process is replaced by a set of random variables $\{\xi_i(\theta)\}$ which is statistically independent of the random variables for the other processes. This procedure thus replaces all the random quantities in the problem, be they random processes or random variables, by a set of uncorrelated Gaussian random variables, that will be denoted by $\{\xi_i(\theta)\}$.

Stochastic Representation of Dynamic Response

The covariance function of the solution process is not known *a priori*, and therefore its Karhunen-Loève expansion cannot be used to represent it. Since the solution process is a function of the material properties, the entries of the nodal response vector can be formally expressed as a nonlinear functional of the set $\{\xi(\theta)\}$ used to represent the stochasticity. It has been shown (Wiener 1938; Cameron and Martin 1947; Kallianpur 1980) that this functional dependence can be represented in terms of polynomials in gaussian random variables, referred to as polynomial chaos. The expansion takes on the following form,

$$u(\mathbf{x},t,\theta) = \sum_{j=0}^{p} u_j(\mathbf{x},t)\Psi_j(\theta) \tag{7}$$

where $\{\Psi_i(\theta)\}$ denotes the set of multidimensional Hermite polynomials in the set $\{\xi_i(\theta)\}$ of basic random variables. These polynomials are orthogonal with respect to the gaussian measure and have zero mean, except for the zero-order polynomial, which is defined as, $\Psi_0 = 1$. A complete probabilistic characterization of the solution process $u(\mathbf{x},t,\theta)$ is obtained once the deterministic coefficients $u_i(\mathbf{x},t)$ have been calculated. A given truncated series can be refined along the random dimension either by adding more random variables to the set $\{\xi_i(\theta)\}$ or by increasing the maximum order of polynomials included in the polynomial chaos expansion. The first refinement takes into account higher frequency random fluctuations of the underlying stochastic process, while the second refinement captures strong nonlinear dependence of the solution process on this underlying process.

Using the orthogonality property of the polynomial chaos, the coefficients in the expansion of the solution process can be computed according to the following equation,

$$u_k(\mathbf{x},t) = [\langle \Psi_k(\theta)u(\mathbf{x},t,\theta)\rangle]/[\langle \Psi_k^2(\theta)\rangle], \quad k=1,\ldots,K \tag{8}$$

Thus, given realizations of the solution process $u(\mathbf{x},t,\theta)$, the coefficients for a polynomial fit according to Eq. (7) can be obtained via statistical averaging as specified by Eq. (8). This procedure will be used in the sequel. Following the polynomial fit to the solution process, additional realizations of the solution can be obtained in a very efficient manner.

Convergence Acceleration

One of the key factors for obtaining an efficient numerical implementation of the stochastic approach based on the polynomial chaos expansion is related to the computation of the statistical average in the numerator of Eq. (8). Clearly, any multidimensional integration rule can be applied to the evaluation of this quantity. In the present work, a stratified sampling technique is used to this end. Both the number of coefficients to be computed using Eq. (8) as well as the number of integration points, or equivalently the number of simulated samples, is directly related to the amount by which the solution process deviates from a Gaussian process. Indeed, the polynomial chaos expansion represents nonGaussian processes as multidimensional Hermite polynomials in Gaussian random variables. Thus the closer the original process is to a Gaussian process, the fewer terms are required in its representation with a preset accuracy. Convergence acceleration can thus be achieved by transforming the original process to a near-Gaussian process, via some judiciously chosen nonlinear transformation, performing the polynomial chaos decomposition on the new process, and then transforming the expanded process back via the inverse nonlinear transformation. It is well known, for example, that structural response peaks as well as other processes associated with extremes of dynamic response, are positive processes with an extreme value probability density function (Grigoriu 1995). It is noted that such an extreme-value distribution relates to the normal distribution through an exponential transformation. Therefore, a logarithmic transformation is applied at the level of the extreme response process before decomposing it into a polynomial chaos expansion. Then the expansion is performed in a transformed space for which the corresponding process is closer to a normal process. Finally, the non-normal process is evaluated using an inverse transformation. This transformation is expressed mathematically by

$$u=\exp\left(\sum_{i=1}^{n}(\langle\ln u\Psi_i\rangle)/(\langle\Psi_i^2\rangle)\Psi_i\right) \tag{9}$$

This significantly speeds up the convergence and improves the accuracy of the computed series expansion for extreme-value responses.

Probabilistic Modeling of Dynamic Soil-Structure Interaction

In this section, it is first demonstrated how the expansions presented in the previous section are implemented into a weighted residual scheme to evaluate an expansion for the solution process. This is followed by a description of the stochastic models used for the various random fields that enter into the description of the problem.

Stochastic Finite Elements

The complex stiffness matrix, $\mathbf{C}(\omega)$, depends on the material properties of the soil, as well as, on the vertical profile of the soil formation. These quantities are modeled as random processes each of which is expanded in its own Karhunen-Loève representation as a linear combination of Gaussian random variables with deterministic spatial functions. The problem is therefore completely determined in terms of a denumerable set of basic random variables. The matrix $\mathbf{C}(\omega)$, being a nonlinear function of these parameters can thus be expanded in terms of the polynomial chaos expansion as follows,

$$\mathbf{C}(\omega)=\sum_{i=0}^{K}\Psi_i\mathbf{C}^i(\omega) \tag{10}$$

where $\mathbf{C}^i(\omega)$ denotes deterministic matrices that can be evaluated given the functional dependence of the stiffness matrix $\mathbf{C}(\omega)$ on the basic random processes such as shear wave velocity and soil profile. A noted feature of the present expansion is its global character, whereby the addition of more terms in the expansion improves not only the behavior near the mean of the probability distribution, but also the resolution around the tail area. Substituting the above expansion into the equation of motion, written in the frequency domain, substituting a polynomial chaos representation of the solution vector, multiplying through by Ψ_l, and then averaging, results in the equation,

$$\sum_{k=0}^{p}\sum_{j=0}^{K}(\Psi_j,\Psi_k,\Psi_l)\begin{bmatrix}\mathbf{C}_{ss}^i & \mathbf{C}_{si}^j\\ \mathbf{C}_{is}^j & (\mathbf{C}_{ii}^j-\mathbf{C}_{ff}^j+\mathbf{x}_{ff}^j)\end{bmatrix}\begin{Bmatrix}\mathbf{u}_s^k\\ \mathbf{u}_f^k\end{Bmatrix}$$

$$=\begin{Bmatrix}\langle\Psi_l\mathbf{P}_s\rangle\\ \langle\Psi_l\mathbf{P}_f\rangle\end{Bmatrix},\quad l=0,\ldots,p \tag{11}$$

The operation leading up to this last equation can be construed as forcing the approximation error to be orthogonal to the basis used in the approximation.

The foregoing analysis involves the solution of a system of linear equations the size of which is equal to the number of degrees of freedom of the system times the number of terms ($p+1$) used in expanding the solution process with respect to the polynomial chaos basis. For most practical problems, the size of this system is too large to handle without customized software. As the present study aims at utilizing commonly available software packages for the PRA, a novel implementation of the stochastic finite element formulation is developed that is theoretically equivalent to the one described above. Thus, for each realization of the set $\{\Psi_i\}$ representing the material properties, the matrix $\mathbf{C}(\omega)$ and the right-hand side vector are assembled. Upon solving the system, a realization of the solution process is thus obtained. This solution is multiplied by each of the Ψ_i and Eq. (8) is evaluated, thus leading to an estimate of the coefficients $\mathbf{u}_k$ in the expansion of the solution process. The stratified sampling technique with pairing control is used for efficiently conducting the simulation procedure (Iman and Conover 1982). After the approximation of the solution in the form given by Eq. (7) has been obtained, additional simulations of the solution can be readily obtained upon simulating the random variables $\{\Psi_i\}$ associated with material properties and random input motion.

Earthquake Motion

The earthquake ground acceleration is represented by a segment of a nonstationary random process. Nonstationarity is introduced by using a deterministic trapezoidal intensity shape function The frequency content of earthquake motion is described locally, at a point on the ground surface, usually by either a probabilistic acceleration response spectrum or a power spectral density function. In the following, a response spectrum specification will be assumed. The three earthquake motion components are assumed to be statistically independent and the frequency-dependent spatial correlation structure of the ground motion field is defined by a coherency spectrum matrix. No attempt is made in the present work to address the issue of wave propagation in a random medium, or that of randomness in the angle of incidence. It is implicitly assumed that the uncertainties introduced by these factors can and have been lumped into the uncertainties used to model the local ground motion.

Local Description of Ground Motion

In engineering practice, probabilistic site-specific ground response spectra are typically defined for hazardous facilities Newmark and Hall 1982; EPRI 1989; Dunbar and Charlwood 1991; LLNL 1993. These probabilistic ground spectra are usually specified by three spectral response curves computed at the 16, 50, and 84% nonexceedance probability levels, corresponding to the median plus or minus one standard deviation from a lognormally distributed amplification of the ground motion. These amplification factors are assumed to be uniform over the whole frequency range and depend only on the level of damping in the system. The present analysis allows these spectra to feature frequency-dependent fluctuations modeled as lognormal stochastic processes. The lognormal random field modeling the spectral amplitudes is represented by its transformed Karhunen-Loève expansion as described earlier. Denoting this process by $S(\omega)$, it is therefore represented as

$$S(\omega)=\exp\left[\sum_{i=0}^{N_S}g_i(\omega)\xi_i\right] \tag{12}$$

where ξ_i=uncorrelated Gaussian variables g_i are as indicated in Eq. (6), and N_s is the number of terms retained in the expansion of the local ground motion. The covariance function used to model the correlation of this process at different frequencies is given by

$$R_{SS}(\Delta\omega) = \sigma_S^2 e^{-(\Delta\omega/b_S)^2} \tag{13}$$

where b_S and σ_S refer to the correlation length and the standard deviation of the process, respectively. It should be noted that as used in the present context, a correlation function represents merely a statistical parametric fit of observed data and does not have any physical interpretation beyond the statistical one. Once a parametric model for the correlation function has been chosen, the parameters of that model can be calibrated to the observed data. In the present case, the quantities b_S and σ_S are evaluated so that the bandwidth of the simulated spectra matches that of the amplification factor of the local soil deposit. Under certain conditions, analytically derived functional forms of the correlation function may be based on the random vibration theory, therefore enhancing the value of the analysis (Der Kiureghian 1981). Realizations of the stochastic process $S(\omega)$ can thus be obtained from corresponding realizations of the set of random variables $\{\xi_i\}$. With each of these realizations of $S(\omega)$, a different ensemble of ground motion time histories, $f(t)$, can be generated according to well established procedures (Levy and Wilkinson 1976). This mapping of $S(\omega)$ into $f(t)$ is assumed to be deterministic. Thus, to each set of random variables $\{\xi_i\}$, is associated one realization of the process $f(t)$, or equivalently, of its Fourier amplitudes, $F(\omega)$. Clearly, processes representing the ground motion and the local soil properties should be correlated. In the present study, however, these two quantities are modeled as independent stochastic processes. As additional data is collected and assimilated into statistical models of ground motion and site conditions, this restriction can be lifted. The relaxation of the independence condition presents no theoretical difficulties, and can be readily implemented once a meaningful model has been postulated.

Spatial Variation of Ground Motion

Wave propagation effects and scattering due to site heterogeneities induce spatial variations in the ground motion, resulting in a loss of coherence. For an incoherent wave field, the unlagged coherence, $\mathrm{Coh}_{U_{i,k}}(\omega)$, for motions at two points *i* and *k*, can be defined as (Abrahamson 1990),

$$\mathrm{Coh}_{U_{i,k}}(\omega) = \mathrm{Coh}_{i,k}(\omega) A(i\omega, X_i - X_k) \exp(i\omega(X_i - X_k)/V_{i-k}) \tag{14}$$

where $\mathrm{Coh}_{i,k}(\omega)$ is the lagged coherence (also known as the coherence), representing the fraction of the total power of seismic motion which can be idealized by a single deterministic plane wave motion referred to as the coherent motion. The coherence does not account for the wave passage effect. In Eq. (14) $A(i\omega, X_i - X_k)$ is a decaying function of ω with unit value at zero frequency. This function models the distribution over frequency of the power of the wave field, modeled as a plane wave. The term $\exp[i\omega(X_i - X_k)/V_{i-k}]$ represents, in the frequency domain, the phase angle between the ground motion at the two points due to the wave passage effect. Moreover, the parameter V_{i-k} is the apparent seismic wave velocity between the two points. For a wave field that is perfectly described by a single plane wave, the function $A(i\omega, X_i - X_k)$ is identically equal to one. The incoherency effect is significantly larger for higher frequency components than for lower frequency components. The effect of incoherence is to reduce the translational motion components and increase the rotational motion components (Ghiocel et al. 1995; Ghiocel 1996a; Ghiocel et al. 1996).

Based on experimental evidence from various records of past earthquakes, a number of analytical forms for the coherence function have been considered (Abrahamson 1990; Zerva and Zhang 1997). In the following, a simplification (Luco and Wong 1986) of a theoretical model based on wave propagation in random media (Uscinski 1977) will be used. It is given by the following equation,

$$\mathrm{Coh}_{i,k}(\omega) = \mathrm{Coh}(|X_i - X_k|, \omega) = \exp(-(\gamma\omega(|X_i - X_k|/V_s)^2) \tag{15}$$

in which γ is the coherence parameter and V_s is the shear wave velocity in the soil. Experimental evidence suggests values of the coherence parameter, γ, in the range 0.1 to 0.3.

A unit variance stochastic process, $c(x,\omega)$ can thus be defined such that its covariance function is given by the coherence $\mathrm{Coh}_{i,k}(\omega)$. This process represents the spatial variability of the ground motion relative to some reference spatial location. The ground motion can be obtained from this process upon multiplying by the local ground motion described in the previous subsection, and accounting for wave passage effects. Moreover, assuming this normalized process to be Gaussian, its Karhunen-Loève expansion, takes on the following simple form, where ξ_i denote independent normalized Gaussian variables,

$$c(x,\omega) = \sum_{i=0}^{N_c} \xi_i c_i(x,\omega) \tag{16}$$

Neglecting the wave passage effect, the Fourier amplitude at frequency ω and *a* distance X_k from the control point at which the local ground Fourier amplitude $F(\omega)$, is being computed, can thus be evaluated according to the expression,

$$F_k(\omega) = \left(\sum_{i=0}^{N_c} \xi_i c_i(X_k, \omega) \right) F(\omega) \tag{17}$$

Given the Fourier amplitudes at the control point, the amplitudes at all points on the soil-structure interface can thus be generated. It should be noted that truncating the summation in Eq. (17) at the N_c term results in the variance of the expansion being somewhat smaller than the variance of the target process which, according to Eq. (15), is equal to 1. This discrepancy, results in the summation in Eq. (17) not being equal to one when $X_k = 0$. This approximation error is treated in this paper by renormalizing the expansion and dividing each term in it by the variance of the approximating process.

Probabilistic Modeling of Material Properties

Soil Properties

Soil properties are considered to be homogeneous in a horizontal plane and modeled as one-dimensional random fields with random fluctuations in the vertical direction. Specifically, the randomness in the dynamic properties of the soil are introduced through the variability in its shear modulus and hysteretic damping. The soil shear modulus at low strains, G_0, is idealized as a one-dimensional lognormal random field in the vertical direction having a nonstationary mean and an assumed correlation function. This idealization is considered to be significantly more realistic and less conservative than the assumption of perfect correlation currently applied for parametric deterministic SSI studies. Fig. 7 shows a number of realizations of this process. For soil layering including different materials, a set of multiple random fields may be considered. Moreover, the shape of the shear modulus-shear strain curve, $G(\gamma)/G_0$ vs γ, is modeled by a random field along the shear strain axis with a nonstationary mean. The mean curve

Table 1. Parameters for Probabilistic Characterization of Problem

Process	Description	PDF	Number of terms in KL expansion	Coefficient of variation	Correlation length
$S(\omega)$	Ground response spectrum	Lognormal	40	0.45	0.85 Hz
$G_0(z)$	Low-strain shear modulus	Lognormal	10	0.4	7 ft
G/G_0	Shear modulus vs. Shear strain	Gaussian	3	0–0.4	2.8
$D_0(z)$	Low-strain damping modulus	Lognormal	10	0.4	7 ft
D/D_0	Damping modulus vs. Shear strain	Gaussian	3	0–0.4	2.8
$c(x,\omega)$	Coherency	Gaussian	4	-	$\gamma=0.2$ $V_s=$ 1000 ft/s
E	Young's modulus for structure	Gaussian	1	0.08	∞ (random variable)
ζ	Material damping ratio in structure	Gaussian	1	0.25	∞ (random variable)

is assumed to have an arbitrary shape which is either introduced by the user or by default stored in the program database. The same modeling assumption used for the shear modulus curve is also implemented for the hysteretic damping-shear strain curve, $D(\gamma)$. All of the above random soil properties are decomposed into their Karhunen-Loève expansion according to

$$G_0(z)=\exp\left[\sum_{i=0}^{N_{G_0}}\xi_i G_{0i}(z)\right] \tag{18}$$

$$G(\gamma)/G_0=\sum_{i=0}^{N_G}\xi_i G_i(\gamma) \tag{19}$$

With similar equations for $D_0(z)$ and $D(\gamma)$. Realizations of $G(\gamma)$ can be obtained by simply multiplying realizations of $G(\gamma)/G_0$ with those of G_0. A correlation function of the following form is used for all these processes

$$R_{GG}(\Delta\gamma)=\sigma_G^2 e^{-(\Delta\gamma/b_G)^2} \tag{20}$$

where $\Delta\gamma$ denotes separation distance along the shear strain dimension, σ_G denotes the standard deviation of the process, and b_G denotes a measure of its correlation length. Given the physically-motivated requirement of the process $G(\gamma)$ to be equal, with probability one, to G_0 at $\gamma=10^{-4}\%$, the coefficient of variation of $G(\gamma)/G_0$ is assumed to have zero variance at this value of γ. Figs. 8 and 9 show realizations of these processes obtained through a Karhunen-Loève synthesis. Clearly, the larger the standard deviation of the assumed processes, the larger the amplitudes of the fluctuation in these processes. Similarly, the smaller the correlation length, the higher the frequency of the oscillations in the realizations. In the present study, the correlation length, b_G, is selected large enough so that the shear modulus and damping are almost always (with high probability) monotonic functions of the shear strain. It is instructive to note that experimental investigations suggest the possibility of a nonmonotonic dependence of dynamic shear and damping moduli on shear strain (Seed et al. 1985; Sun et al. 1988; Vucetic and Dobry 1991; Mitchell 1993). Such behavior can be easily reproduced by the present stochastic model by using a smaller correlation length in the corresponding correlation functions.

At this stage of the analysis, $N_S+N_C+N_{G_0}+N_{D_0}+N_G+N_D$ random variables characterize the uncertainty of the dynamical system.

Structural Properties

Structural damping and stiffness parameters are assumed to be random variables. This assumption is based on the fact that the

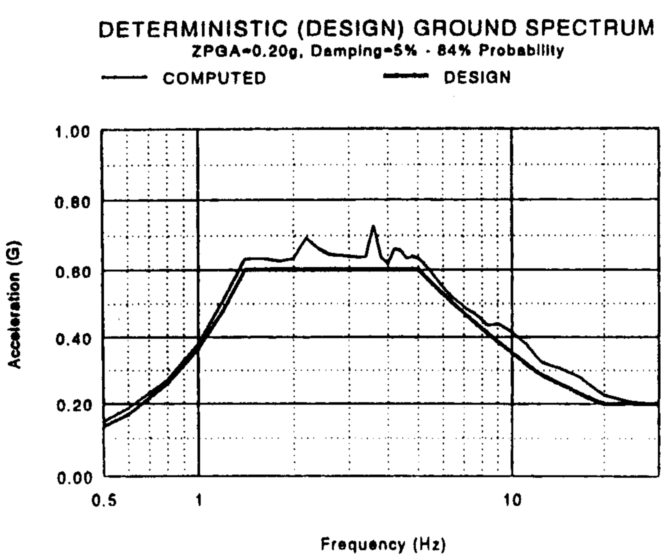

Fig. 3. Computed versus design ground response spectra

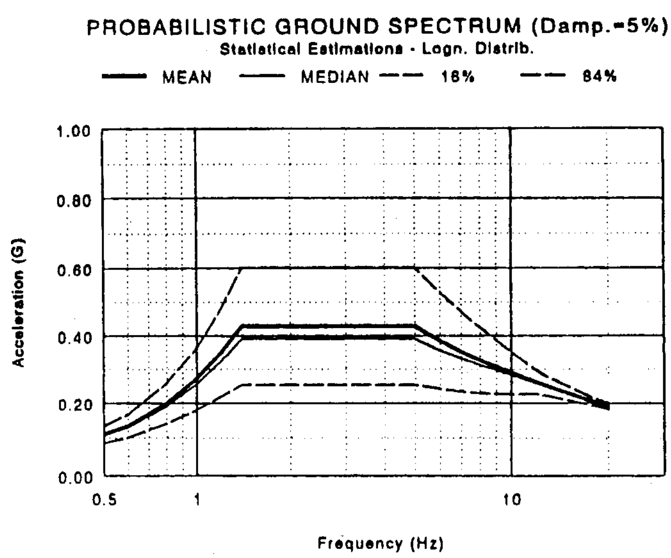

Fig. 4. Probabilistic ground response spectra

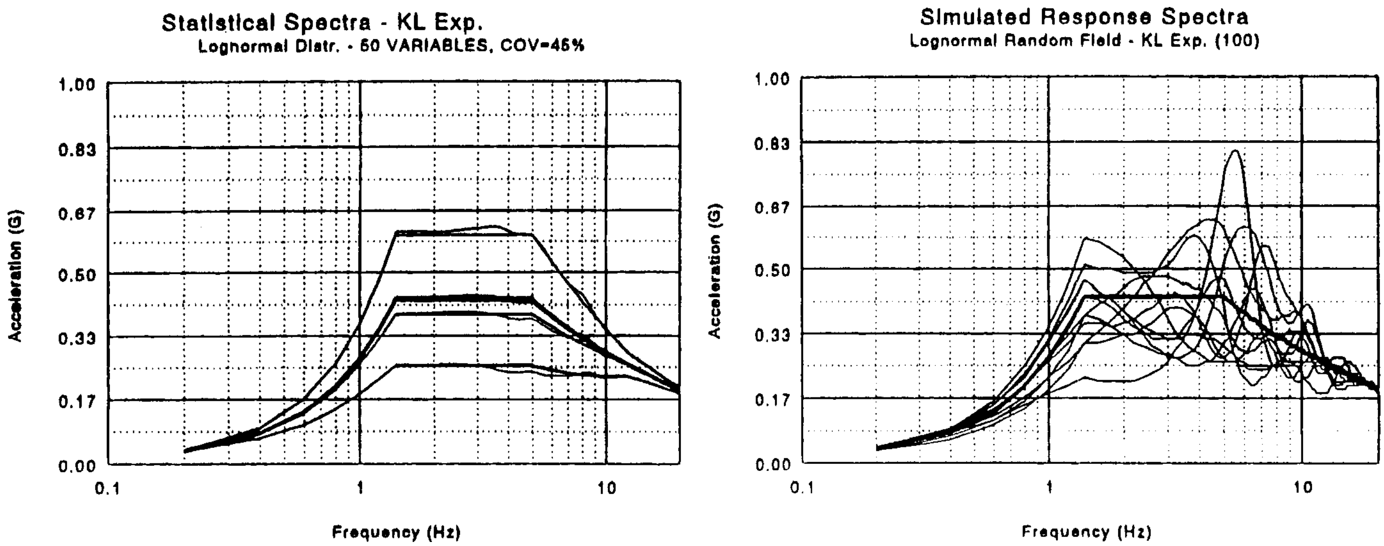

Fig. 5. KL-simulated probabilistic ground response spectra

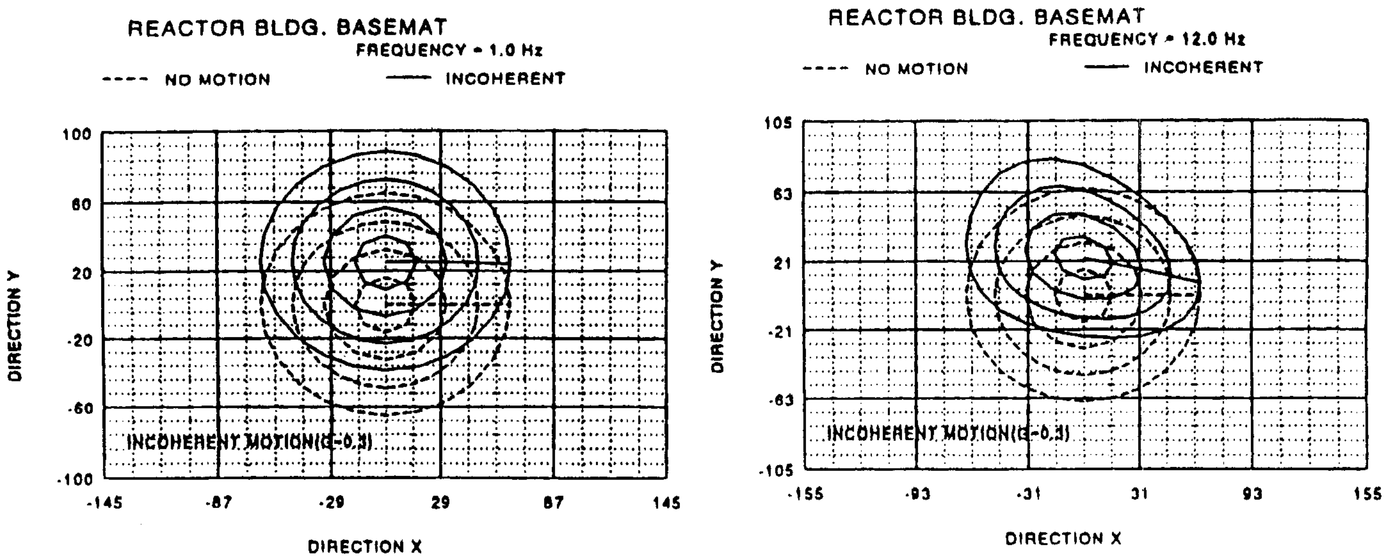

Fig. 6. Shapes of incoherent motion at soil-structure interface at two frequencies

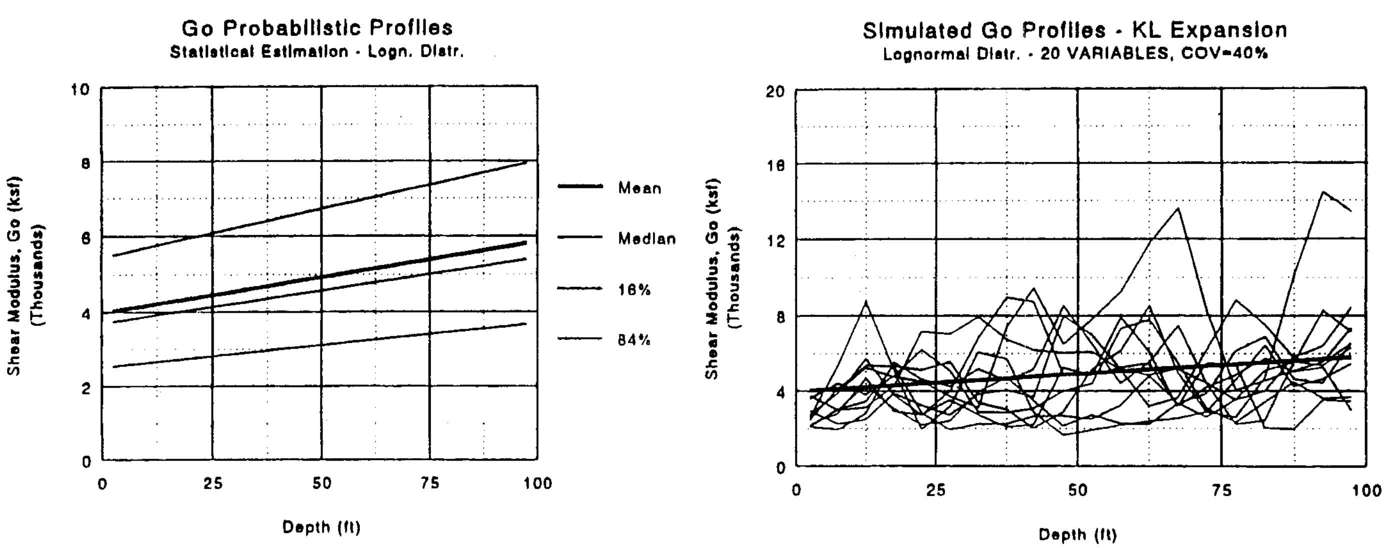

Fig. 7. Statistical estimates and realizations of probabilistic low-strain shear modulus profiles

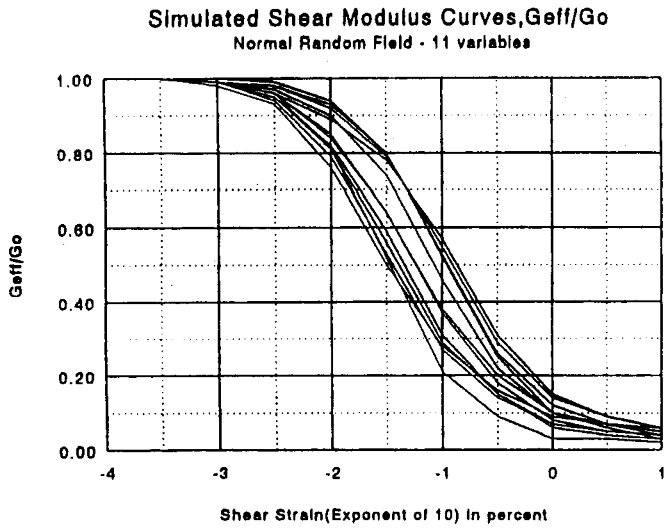

Fig. 8. Realizations of shear modulus versus shear strain curve

random variation of these parameters within the superstructure are appropriately described by a discrete random field or a set of independent random variables rather than a continuous random field expandable in a Karhunen-Loève series. In order to unify the treatment of random variables and random processes, the former are viewed as special cases of the latter with a very large correlation length in which case a single term in their Karhunen-Loève expansion is enough to capture the significant portion of the variability.

Evaluation of An Actual Design of A Reactor Building

A typical reactor building (Lysmer et al. 1988) subjected to earthquake motion is investigated using both a probabilistic and a deterministic analysis. For the probabilistic analysis the proposed approach is applied, while for the deterministic analysis the current design practice is considered. The finite element model used for seismic soil-structure interaction analysis is shown in Fig. 1.

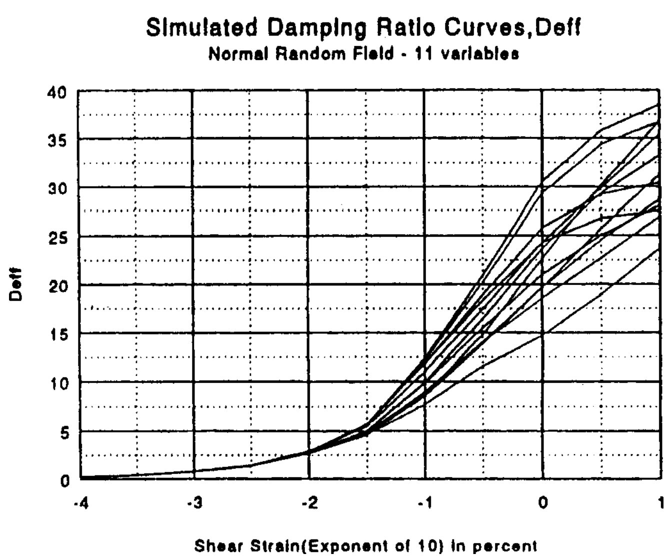

Fig. 9. Realizations of damping versus shear strain curve

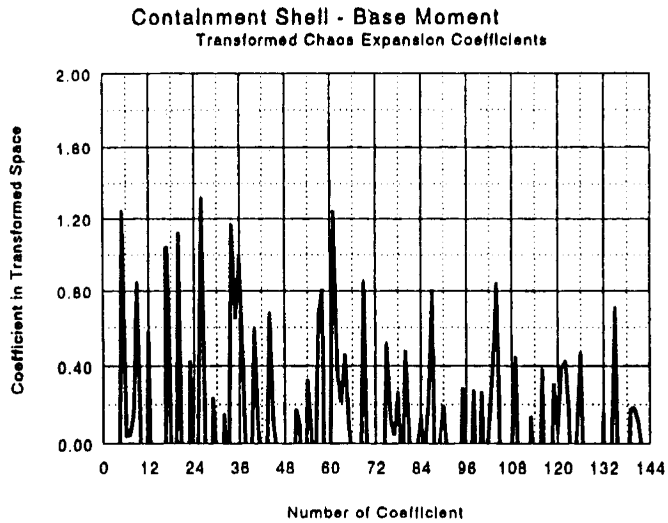

Fig. 10. Coefficient of polynomial chaos expansion of the stochastic base bending moment solution

This computational model represents a typical breakdown of the problem for seismic SSI calculations of a typical reactor building (Popescu 1995). The superstructure is modeled by beam elements and the basemat is modeled by solid elements. Rigid links are introduced to transmit the rocking motion from the superstructure stick to the basemat. The ACS-SASSI/PC computer code is used for evaluating the seismic soil-structure interaction for both the probabilistic and the deterministic analyses.

Table 1 shows the parameters needed for characterizing each of the random quantities used in this example. The covariance function of all processes is assumed to be of the form given by Eq. 13 with the correlation length shown in Table 1 referring to the parameter b_S in that equation. The total number of basic random variables used in the probabilistic description of the problem is thus equal to 72. Assuming the various processes to be independent, these random variables are themselves independent, and for each joint realization thereof, realizations of the corresponding stochastic processes can be simply computed by synthesizing the Karhunen-Loève (or the modified Karhunen-Loève) expansions, and the corresponding realization of the solution process can be obtained by solving the associated deterministic problem. Thus, a number of realizations of the solution can be efficiently synthesized.

The deterministic analysis is performed for a seismic input defined by the design ground spectrum associated with a 84% probability of nonexceedance. A design spectrum-compatible accelerogram is generated as input to the site response portion of the soil-structure interaction analysis. As shown in Fig. 3 the computed response spectra of the generated accelerogram closely envelope the target design spectrum. The soil properties used in the deterministic analysis are estimated from a database of experimental results. In accordance to the current seismic design requirements, two additional extreme bounds, being half and twice the best-estimate, are also considered. The final results of the deterministic analysis are obtained by enveloping the results for the three soil-structure interaction analyses associated with these three sets of values of soil parameters.

For the probabilistic analysis the earthquake input is defined by a probabilistic response ground spectrum as shown in Fig. 4. The four spectral curves correspond to the mean, the median, 16 and 84% nonexceedance probability estimates. A lognormal prob-

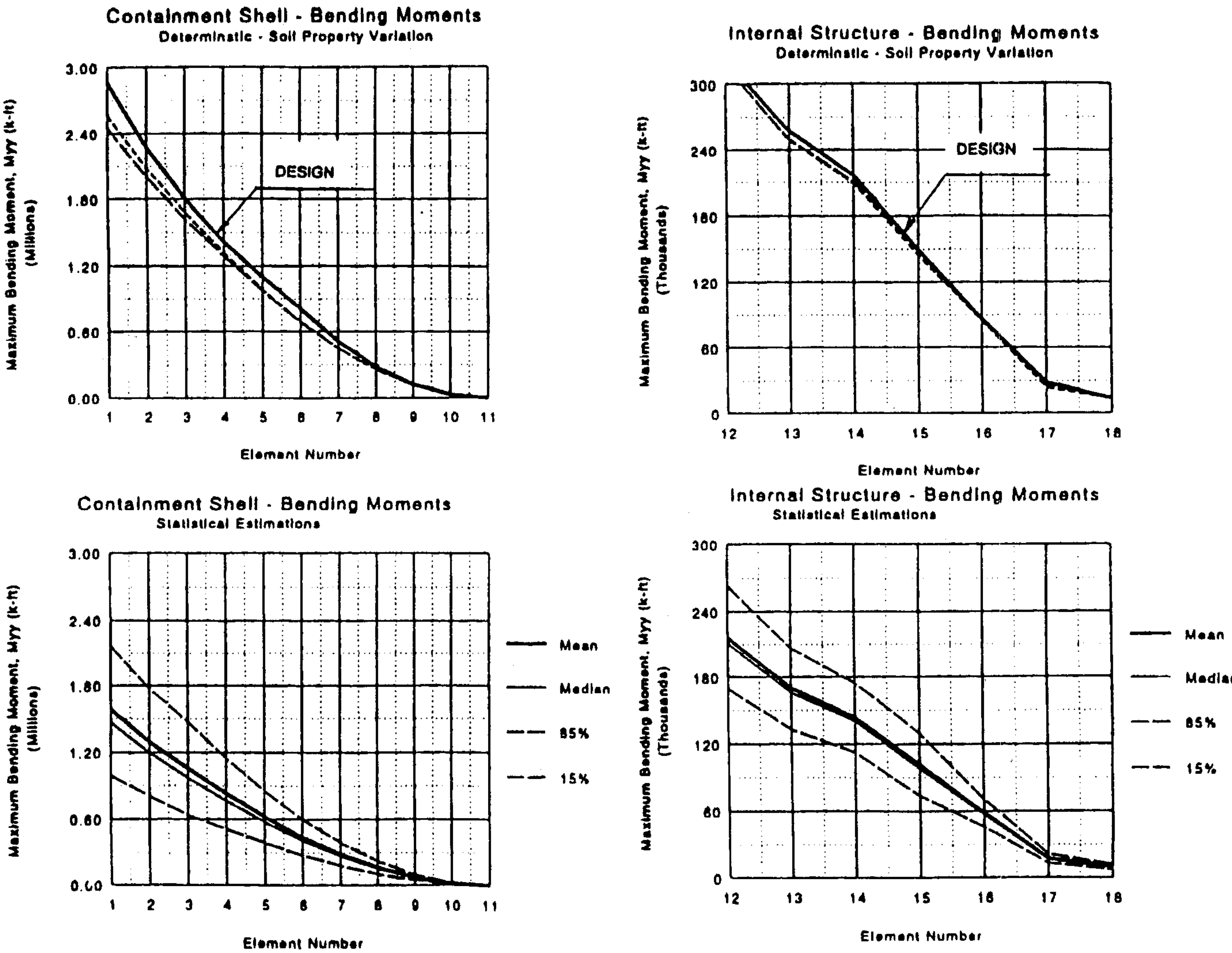

Fig. 11. Comparison of deterministic and probabilistic bending moment in reactor building structure

ability density function is assumed in specifying these nonexceedance probabilities. Realizations of this lognormal field are synthesized by relying on the modified Karhunen-Loève expansion previously described. The correlation length along the frequency axis is selected depending on the desired bandwidth of simulated spectra, a function of the damping level. Fig. 5 illustrates the ensemble statistics (for nonexceedance probabilities of 16, 50, 84% and mean) obtained from a statistical population of ground response spectra of size 100, along with a few realizations from that population.

For the probabilistic SSI analysis the effect of motion incoherency is considered using a Luco-Wong model with a parameter of 0.20. The coherency matrix of the random motion field is decomposed using Karhunen-Loève expansion. The amplitude shapes of the incoherent motion at the soil-basemat interface at 1 and 12 Hz frequency are shown in Fig. 6. It should be noted from the figure that, as expected, the incoherency increases significantly with frequency.

Probabilistic soil properties are defined assuming that the low-strain soil shear modulus and hysteretic damping profiles (variation with depth) are lognormal random fields. Fig. 7 shows the probabilistic shear modulus profile (statistically estimated profiles are included). Plotted curves correspond to mean, median and 16 and 84% nonexceedance probabilities. The transformed space Karhunen-Loève expansion is used to represent these fields. The variation of nondimensional shear modulus and hysteretic damping versus shear strain are modeled as normal random fields. The Karhunen-Loève expansion is again employed. Realizations of the shear modulus and damping profiles against shear strain are plotted in Figs. 8 and 9.

Probabilistic structural properties are described using random variables. Specifically, the Young's elastic modulus and the material damping ratio are assumed to be independent normal random variables, each having a coefficient of variation of 0.25.

Fig. 10 shows the coefficients of the transformed polynomial chaos expansion using 72 basic random variables. Between 1 and 71 are the coefficients of the first-order polynomials, and between 73 and 144 are the coefficients of the second-order polynomials. The second-order terms featuring coupling between the ξ_i have been neglected in the present analysis. It is noted that less than half of the number of basic random variables have significant contributions. It is very difficult, however, for the complex soil-structure interaction problem at hand, to identify *a priori* the most significant variables. Additional insight along those lines could greatly enhance the efficiency of the proposed analysis.

Figs. 11 and 12 show a comparison between deterministic and probabilistic analysis results, both in terms of floor spectra and structural forces. The probabilistic estimates are obtained from

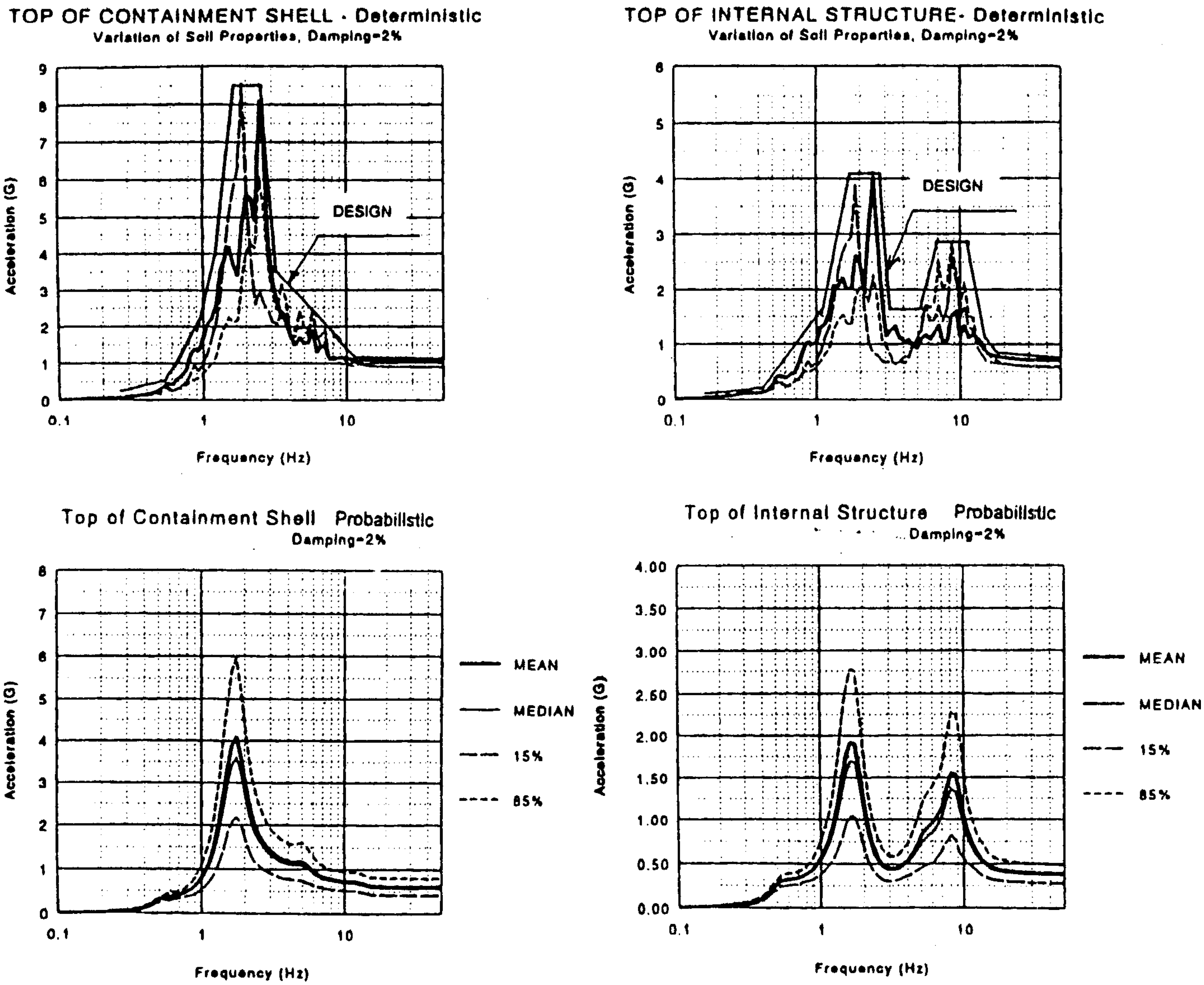

Fig. 12. Comparison of deterministic and probabilistic in-structure spectra in reactor building structure

5,000 realizations synthesized from the polynomial chaos expansion of the solution process given by Eq. (9). The deterministic results correspond to a low-strain shear modulus specified by its nominal value, twice the nominal value, and half that value. These bounds are associated with a low nonexceedance probability levels of 0.001, and are consistent with current design practice. The stochastic analysis accounts for all the random quantities as described above. It appears from these results that the current design practice is overly conservative by not providing for the variability in parameters other than G_0.

Conclusions

The paper presents a novel stochastic approach for seismic soil-structure interaction problems. The proposed approach based on polynomial chaos representation of stochastic solution offers accuracy, efficiency, and significant modeling advantages in comparison with the current risk assessment methodologies. The proposed stochastic approach addresses efficiently problems involving a large number of variables, such is the case with the dynamic SSI problem, while allowing for the efficient treatment of random field. These are useful for the idealization of the dynamic loading and system parameters. In addition, the novel stochastic approach is capable of handling large variability and highly nonlinear problems. The proposed approach is currently being extended for probabilistic modeling of strength capacity of structural elements, with the final scope of creating an integrated computational tool for an accurate structural risk assessment of hazardous facilities including the soil-structure interaction effect.

Acknowledgments

The financial support of the National Science Foundation through the SBIR and Geomechanics programs under Grant Nos. DMI-0966321, CMS-9596238, and CMS-9870005 is gratefully acknowledged.

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